

# $AdS_4 \times \mathbb{CP}^3$ superstring in the light-cone gauge

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## Abstract

The Type IIA superstring action on the  $AdS_4 \times \mathbb{CP}^3$  background, obtainable by the double dimensional reduction of the  $AdS_4 \times S^7$  supermembrane, is considered in the  $\kappa$ -symmetry light-cone gauge, in which the light-like directions are chosen on the  $D = 3$  Minkowski boundary of  $AdS_4$ . Such choice of the gauge condition relies on representing the  $AdS_4 \times S^7$  background isometry superalgebra  $osp(4|8)$  (and correspondingly the  $osp(4|6)$  isometry superalgebra of the  $AdS_4 \times \mathbb{CP}^3$  background) as  $D = 3$  extended superconformal algebra. The gauge-fixed action includes contributions up to the 4th power in the fermions.

## 1 Introduction

Proposed by Aharony, Bergman, Jafferis and Maldacena (ABJM) [1] the duality relation between the  $D = 3$   $\mathcal{N} = 6$  superconformal Chern-Simons-matter theory with the  $U(N) \times U(N)$  gauge symmetry and level  $k$  and M-theory on the  $AdS_4 \times (S^7/\mathbf{Z}_k)$  background provided the novel instance of the gauge fields/strings correspondence. Whenever  $N, k \rightarrow \infty$  with the t'Hooft coupling  $\lambda = N/k$  fixed the dual theory reduces to the IIA superstring on the  $AdS_4 \times \mathbb{CP}^3$  background. So one of the pivotal problems in exploring the ABJM duality is to construct and quantize the string theory on  $AdS_4 \times \mathbb{CP}^3$ . The problem of constructing the superstring action on  $AdS_4 \times \mathbb{CP}^3$  including the fermions has been addressed in [2]-[5]. In Ref. [2] there was applied the supercoset approach previously proposed to obtain the Green-Schwarz (GS) superstring action on the  $AdS_5 \times S^5$  superbackground [6], [7]. The basic observation is that the symmetry group  $SO(2, 3) \times SU(4)$  of the bosonic background can be viewed as the bosonic subgroup of the  $OSp(4|6)$  supergroup that also includes 24 fermionic generators. The action constructed in [2] using the  $OSp(4|6)/(SO(1, 3) \times U(3))$  supercoset element includes 10 bosonic and 24 fermionic<sup>2</sup> degrees of freedom, is invariant under the 8-parameter  $\kappa$ -symmetry transformations and is classically integrable. Such supercoset action corresponds to the complete superstring action on  $AdS_4 \times \mathbb{CP}^3$  with 32 fermions [5], in which the  $\kappa$ -symmetry gauge freedom has been partially fixed. Since  $S^7$  admits the Hopf fibration representation as  $\mathbb{CP}^3 \times U(1)$  [8], [9] the complete superstring action on  $AdS_4 \times \mathbb{CP}^3$  can be constructed by performing the double dimensional reduction [10] of the membrane action on the  $D = 11$  maximally supersymmetric background  $AdS_4 \times S^7$  [11] that can also be viewed as the supercoset manifold  $OSp(4|8)/(SO(1, 3) \times SO(7))$ . This complete action on  $AdS_4 \times \mathbb{CP}^3$  describes all possible superstring motions and allows a wider choice of the  $\kappa$ -symmetry gauges compared to the supercoset action. However, whether it is integrable remains unclear.

Among possible  $\kappa$ -symmetry gauge conditions a special role is played by the light-cone type gauges. In flat superspace it is the light-cone gauge in which the GS equations of motion linearize and the model can be straightforwardly quantized. Light-cone type gauge conditions

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<sup>2</sup>Equal in number to the Type IIA supersymmetries preserved by the  $AdS_4 \times \mathbb{CP}^3$  background.

also appear to be useful in exploring the string theory on  $AdS_5 \times S^5$  (see [12] for review and references). The superstring model on  $AdS_4 \times \mathbb{CP}^3$ , as well as its Penrose limit have been studied in the light-cone type gauges in [13]–[19]. Here we consider the superstring action on the  $AdS_4 \times \mathbb{CP}^3$  background beyond the  $OSp(4|6)/(SO(1,3) \times U(3))$  supercoset approach in the fermionic light-cone gauge corresponding to the choice of the light-like directions within the  $D = 3$  Minkowski boundary of  $AdS_4$ <sup>3</sup>. Similarly to our previous paper [21] we elaborate on the presentation of the  $AdS_4 \times S^7$  background isometry superalgebra  $osp(4|8)$  as the  $D = 3$  extended superconformal algebra and the transformation of the Cartan forms to the superconformal basis<sup>4</sup>. To make the exposition more self-contained we start by reviewing the  $OSp(4|8)/(SO(1,3) \times SO(7))$  supercoset membrane action and its dimensional reduction to the superstring action on  $AdS_4 \times \mathbb{CP}^3$ . The details of the notation, spinor and isometry algebras are taken to the Appendixes.

## 2 The supermembrane on $AdS_4 \times S^7$ and its reduction to the $D = 10$ IIA superstring on $AdS_4 \times \mathbb{CP}^3$

The  $D = 11$  supermembrane action on the  $AdS_4 \times S^7$  background [11] is given by

$$S = - \int_V d^3\xi \sqrt{-g^{(3)}} + S_{WZ}, \quad (1)$$

where  $g^{(3)}$  is the determinant of the induced world-volume metric

$$g_{\underline{i}\underline{j}}^{(3)} = E_{\underline{i}}^{\hat{m}} E_{\underline{j}\hat{m}} \quad (2)$$

and the Wess-Zumino (WZ) term

$$S_{WZ} = s \int_{\mathcal{M}_4} H_{(4)} \quad (3)$$

is presented as the integral of the closed 4-form

$$H_{(4)} = \frac{i}{2} F^{\hat{\alpha}} \wedge \mathfrak{g}^{\hat{m}\hat{n}}{}_{\hat{\alpha}}{}^{\hat{\beta}} F_{\hat{\beta}} \wedge E_{\hat{m}} \wedge E_{\hat{n}} + \varepsilon_{m'n'k'l'} E^{m'} \wedge E^{n'} \wedge E^{k'} \wedge E^{l'} \quad (4)$$

over the 4-dimensional auxiliary hypersurface  $\mathcal{M}_4$ , whose boundary coincides with the supermembrane world volume  $V$ . The first term has the same structure as in the flat superspace [25], while the second term is the contribution of the nonzero bosonic 4-form of the background. The  $D = 11$  supervielbein bosonic components  $E^{\hat{m}} = (E^{m'}, E^{I'}) = (G^{0'm'}, \Omega^{8I'})$  consist of the  $AdS_4$  and  $S^7$  vielbeins  $G^{0'm'}$  and  $\Omega^{8I'}$  that are the Cartan forms corresponding to the  $so(2,3)/so(1,3)$  and  $so(8)/so(7)$  generators  $M_{0'm'}$  and  $V^{8I'}$  respectively. Together with the fermionic 1-forms  $F^{\hat{\alpha}} \equiv F^{\alpha A'}$  associated with the  $osp(4|8)$  fermionic generators  $O_{\hat{\alpha}} \equiv O_{\alpha A'}$  they satisfy the  $osp(4|8)$  Maurer-Cartan (MC) equations

$$\begin{aligned} dG^{0'm'} &= 2G^{m'}{}_{n'} \wedge G^{0'n'} + \frac{i}{4} F^{\alpha A'} \wedge C_{A'B'} \Gamma_{\alpha\beta}^{m'} F^{\beta B'}, \\ d\Omega^{8I'} &= 2\Omega^{I'J'} \wedge \Omega^{8J'} - \frac{i}{4} F^{\alpha A'} \wedge \Gamma_{\alpha\beta}^5 \gamma_{A'B'}^{I'} F^{\beta B'}, \\ dF^{\alpha A'} &= -\frac{1}{2} G^{m'n'} \wedge F^{\beta A'} \Gamma_{m'n'\beta}{}^{\alpha} - G^{0'm'} \wedge F^{\beta A'} \Gamma_{m'\beta}{}^{\gamma} \Gamma^5{}_{\gamma}{}^{\alpha} \\ &\quad + \frac{1}{2} \Omega^{I'J'} \wedge F^{\alpha B'} \gamma_{B'A'}^{I'J'} - \frac{1}{2} \Omega^{8I'} \wedge F^{\alpha B'} \gamma_{B'A'}^{I'} \end{aligned} \quad (5)$$

<sup>3</sup>Analogous  $\kappa$ -symmetry gauge conditions were proposed in [20] for the  $AdS_5 \times S^5$  superstring.

<sup>4</sup>The splitting of the fermionic generators/coordinates into the supersymmetry/superconformal ones w.r.t. the symmetry of the Minkowski boundary of  $AdS$  was introduced in [22], [23], [24] for the study of brane models on the  $AdS \times S$  backgrounds.

that can be derived by taking into account Eq.(20) below and the  $osp(4|8)$  (anti)commutation relations in the form (100), (110), (116), (118) given in Appendix B. These MC equations are used to show the closeness of  $H_{(4)}$ .

To verify the  $\kappa$ -invariance of the membrane action (1) consider the following variation of the fermionic vielbein components

$$F_{\hat{\alpha}}(\delta_{\kappa}) = \Pi_{\hat{\alpha}}^{\hat{\beta}} \kappa_{\hat{\beta}}(\xi), \quad (6)$$

where the matrix  $\Pi_{\hat{\alpha}}^{\hat{\beta}}$  has the form

$$\Pi_{\hat{\alpha}}^{\hat{\beta}} = \delta_{\hat{\alpha}}^{\hat{\beta}} + \frac{q}{\sqrt{-g^{(3)}}} \varepsilon^{ijk} E_{\underline{i}}^{\hat{m}_1} E_{\underline{j}}^{\hat{m}_2} E_{\underline{k}}^{\hat{m}_3} \mathfrak{g}_{\hat{m}_1 \hat{m}_2 \hat{m}_3 \hat{\alpha}}^{\hat{\beta}}, \quad (7)$$

accompanied by  $E^{\hat{m}}(\delta_{\kappa}) = 0$ . Using that  $dH_{(4)} = 0$  the variation of  $H_{(4)}$  reduces to

$$\delta_{\kappa} H_{(4)} = id \left( F^{\hat{\alpha}} \mathfrak{g}_{\hat{m} \hat{n} \hat{\alpha}}^{\hat{\beta}} F_{\hat{\beta}}(\delta_{\kappa}) \wedge E^{\hat{m}} \wedge E^{\hat{n}} \right). \quad (8)$$

Substituting (6) into (8) and performing the  $\gamma$ -matrix rearrangements using the formulae adduced in [26] yields the expression for the Wess-Zumino 4-form variation

$$\delta_{\kappa} H_{(4)} = id \left( F^{\hat{\alpha}} \mathfrak{g}_{\hat{m} \hat{n} \hat{\alpha}}^{\hat{\beta}} \kappa_{\hat{\beta}} \wedge E^{\hat{m}} \wedge E^{\hat{n}} - \frac{6q}{\sqrt{-g^{(3)}}} \varepsilon^{ijk} \varepsilon^{\underline{i}' \underline{j}' \underline{k}'} (E_{\underline{i}} \cdot E_{\underline{i}'})(E_{\underline{j}} \cdot E_{\underline{j}'})(E_{\underline{k}} \cdot E_{\underline{k}'}) E_{\hat{m} \underline{k}'}^{\hat{\alpha}} F_{\underline{k}}^{\hat{\alpha}} \mathfrak{g}_{\hat{\alpha}}^{\hat{m}} \kappa_{\hat{\beta}}^{\hat{\beta}} \right). \quad (9)$$

It has to be canceled by the  $\kappa$ -variation of the kinetic term of (1) that is defined by the variation of the induced metric determinant  $g^{(3)}$ . Taking into account that the  $\kappa$ -variation of the bosonic vielbein reads

$$\delta_{\kappa} E^{\hat{m}} = -\frac{i}{2} F^{\hat{\alpha}} \mathfrak{g}_{\hat{\alpha}}^{\hat{m}} F_{\hat{\beta}}(\delta_{\kappa}) \quad (10)$$

one obtains

$$\delta_{\kappa} \sqrt{-g^{(3)}} = \frac{i}{4\sqrt{-g^{(3)}}} \varepsilon^{ijk} \varepsilon^{\underline{i}' \underline{j}' \underline{k}'} (E_{\underline{i}} \cdot E_{\underline{i}'})(E_{\underline{j}} \cdot E_{\underline{j}'})(E_{\underline{k}} \cdot E_{\underline{k}'}) E_{\hat{m} \underline{k}'}^{\hat{\alpha}} F_{\underline{k}}^{\hat{\alpha}} \mathfrak{g}_{\hat{\alpha}}^{\hat{m}} \kappa_{\hat{\beta}}^{\hat{\beta}} - \frac{3iq}{2} F^{\hat{\alpha}} \mathfrak{g}_{\hat{\alpha}}^{\hat{m} \hat{n}} \kappa_{\hat{\beta}}^{\hat{\beta}} \wedge E_{\hat{m}} \wedge E_{\hat{n}}. \quad (11)$$

The  $\kappa$ -invariance condition of the action (1) then fixes the values of the numerical coefficients  $s = \pm \frac{1}{4}$ ,  $q = \mp \frac{1}{6}$ <sup>5</sup> so that the final form of the WZ term is given by

$$S_{WZ} = \pm \frac{1}{4} \int_{\mathcal{M}_4} \left( \frac{i}{2} F^{\hat{\alpha}} \wedge \mathfrak{g}_{\hat{\alpha}}^{\hat{m} \hat{n}} F_{\hat{\beta}} \wedge E_{\hat{m}} \wedge E_{\hat{n}} + \varepsilon_{m' n' k' l'} E^{m'} \wedge E^{n'} \wedge E^{k'} \wedge E^{l'} \right). \quad (12)$$

Dimensional reduction of the  $D = 11$  supermembrane action to the  $D = 10$  Type IIA superstring was described for general superbackground in [10]. The vielbeins are chosen not to depend on the reduction direction coordinate  $y \in [0, 2\pi)$ . However, when the bosonic components of the supervielbein receive contribution  $E_y^{\hat{m}'}$  proportional to the differential  $dy$  of the  $U(1)$  coordinate<sup>6</sup> the local Lorentz rotation in the tangent space has to be performed

$$E^{\hat{m}} \rightarrow L^{\hat{m}}_{\hat{n}} E^{\hat{n}}, \quad F^{\hat{\alpha}} \rightarrow L^{\hat{\alpha}}_{\hat{\beta}} F^{\hat{\beta}}, \quad L^{\hat{m}}_{\hat{n}} \in SO(1, 10), \quad L^{\hat{\alpha}}_{\hat{\beta}} \in Spin(1, 10) \quad (13)$$

<sup>5</sup>Such choice of the value of  $q$  makes the matrix  $\Pi_{\hat{\alpha}}^{\hat{\beta}}$  the projector that eliminates half components of the transformation parameter  $\kappa^{\hat{\alpha}}(\xi)$ .

<sup>6</sup>As is the case for the  $OSp(4|8)/(SO(1, 3) \times SO(7))$  supercoset elements considered in [5] and here.

with the parameters determined by  $E_y^{\hat{m}'}$  to bring the supervielbein bosonic components to the Kaluza-Klein ansatz form

$$(LE)^{\hat{m}'} = E^{\hat{m}'}, \quad (LE)^{11} = E^{11} = \Phi(dy + A), \quad (14)$$

where  $E^{\hat{m}'}$  is the bosonic part of the  $D = 10$  supervielbein that does not depend on  $y$  and  $dy$ , the  $D = 10$  IIA superfield  $\Phi = e^{2\phi/3}$  is related to the dilaton  $\phi$ , and  $A$  is the RR 1-form potential. For the fermionic components of the  $D = 11$  supervielbein the reduction ansatz is

$$(LF)^{\hat{\alpha}} = E^{\hat{\alpha}} + e^{-2\phi/3} \chi^{\hat{\alpha}} E^{11} : \quad E^{\hat{\alpha}} = (Lf)^{\hat{\alpha}} - (LF_y)^{\hat{\alpha}} A, \quad \chi^{\hat{\alpha}} = (LF_y)^{\hat{\alpha}}, \quad (15)$$

where  $E^{\hat{\alpha}}$  are the  $D = 10$  supervielbein fermionic components,  $\chi^{\hat{\alpha}}$  is the dilatino superfield and there has been separated the  $dy$ -dependent contribution to the  $D = 11$  supervielbein fermionic components

$$F^{\hat{\alpha}} = f^{\hat{\alpha}} + dy F_y^{\hat{\alpha}}. \quad (16)$$

Upon identification of  $y$  with the world-volume compact direction coordinate the kinetic term of the membrane action (1) reduces to the Nambu-Goto string action in the Kaluza-Klein frame

$$\int_V d^3 \xi \sqrt{-g^{(3)}} \rightarrow \int_{\Sigma} d\tau d\sigma \Phi \sqrt{-g^{(2)}}, \quad (17)$$

where  $g^{(2)} = \det g_{ij}^{(2)}$  is the determinant of the induced world-sheet metric

$$g_{ij}^{(2)} = E_i^{\hat{m}'} E_{\hat{m}'}^j, \quad (18)$$

while the membrane WZ term reduces to the integral of the NS-NS 3-form over the auxiliary 3-dimensional hypersurface  $\mathcal{M}_3$ , whose boundary coincides with the superstring world-sheet  $\Sigma$

$$\int_{\mathcal{M}_4} H_{(4)} \rightarrow \int_{\mathcal{M}_3} H_{(3)}. \quad (19)$$

To specify the above discussion let us describe the transformation of the  $so(8)$  Cartan forms to the  $\mathbb{CP}^3 \times U(1)$ -basis that exhibits the Hopf fibration realization of  $S^7$  necessary to perform the double dimensional reduction and also of the  $sp(4)$  Cartan forms to the  $D = 3$  conformal basis. For the  $OSp(4|8)/(SO(1,3) \times SO(7))$  supercoset element  $\mathcal{G}$  the  $SO(1,3) \times SO(7)$  covariant decomposition of the Cartan forms reads

$$\mathcal{G}^{-1} d\mathcal{G} = G^{m'n'} M_{m'n'} + G^{0'm'} M_{0'm'} + \Omega^{I'J'} V^{I'J'} + \Omega^{8I'} V^{8I'} + F^{\alpha A'} O_{\alpha A'}. \quad (20)$$

The first two terms correspond to the  $D = 3$  conformal sector and after the transformation of the generators to the conformal basis, as described in Appendix B, are brought to the form

$$G^{m'n'} M_{m'n'} + G^{0'm'} M_{0'm'} = G^{mn} M_{mn} + \Delta D + \omega^m P_m + c^m K_m, \quad (21)$$

where

$$\omega^m = G^{0'm} - G^{3m}, \quad c^m = G^{0'm} + G^{3m}, \quad \Delta = -G^{0'3}. \quad (22)$$

The  $AdS_4$  vielbein components, used in constructing the superstring action, in the conformal basis are expressed as

$$G^{0'm} = \frac{1}{2}(\omega^m + c^m), \quad G^{0'3} = -\Delta. \quad (23)$$

The third and the fourth terms in (20) correspond to the  $so(8)$  generators and 1-forms and further decompose as

$$\Omega^{I'J'}V^{I'J'} + \Omega^{8I'}V^{8I'} = \Omega^{IJ}V^{IJ} + \Omega^{78}V^{78} + \Omega^{8I}V^{8I} + 2\Omega^{7I}V^{7I}. \quad (24)$$

To perform the dimensional reduction of the  $AdS_4 \times S^7$  membrane action one is required to change the generator basis to that corresponding to the Hopf fibration realization of the 7-sphere (see Appendix B). Then the two last terms in (24) corresponding to the  $so(8)/(so(2) \times so(6))$  coset generators acquire the form

$$\Omega^{8I}V^{8I} + 2\Omega^{7I}V^{7I} = \Omega_a T^a + \Omega^a T_a + \tilde{\Omega}_a \tilde{T}^a + \tilde{\Omega}^a \tilde{T}_a, \quad (25)$$

where

$$\begin{aligned} \Omega_a &= \Omega^7_{4a} - \frac{i}{2}\Omega^8_{4a}, & \tilde{\Omega}^a &= -\Omega^{74a} - \frac{i}{2}\Omega^{84a}, \\ \tilde{\Omega}_a &= -\Omega^7_{4a} - \frac{i}{2}\Omega^8_{4a}, & \tilde{\Omega}^a &= \Omega^{74a} - \frac{i}{2}\Omega^{84a} \end{aligned} \quad (26)$$

and  $\Omega^{7(8)}_{4a} = \Omega^{7(8)I}\rho^I_{4a}$ ,  $\Omega^{7(8)4a} = \Omega^{7(8)I}\tilde{\rho}^{I4a}$ . The 1-forms

$$\Omega^8_{4a} = i(\Omega_a + \tilde{\Omega}_a), \quad \Omega^{84a} = i(\Omega^a + \tilde{\Omega}^a) \quad (27)$$

will be used in constructing the superstring action. Changing the basis in the  $so(6) \oplus so(2)$  sector that is described by the first two terms in (24) yields

$$\Omega^{IJ}V^{IJ} + \Omega^{78}V^{78} = \tilde{\Omega}_a{}^b \tilde{V}_b{}^a + \tilde{\Omega}_b{}^a \tilde{V}_a{}^b + \Omega_a{}^4 V_4{}^a + \Omega_4{}^a V_a{}^4 + hH, \quad (28)$$

where

$$\begin{aligned} \tilde{\Omega}_a{}^b &= \Omega_a{}^b - \delta_a^b \Omega_c{}^c + \delta_a^b h, & \tilde{\Omega}_b{}^b &= -2\Omega_b{}^b + 3h, \\ \Omega^{78} &= -\tilde{\Omega}_a{}^a - h, \end{aligned} \quad (29)$$

and the  $so(6)$  Cartan forms have been decomposed into the  $u(3)$   $\Omega_a{}^b$  and  $su(4)/u(3)$   $\Omega_a{}^4$ ,  $\Omega_4{}^a$  ones as

$$\Omega_A{}^B = \frac{i}{2}\Omega^{IJ}\rho^{IJ}{}_A{}^B = \begin{pmatrix} \Omega_a{}^b & \Omega_a{}^4 \\ \Omega_4{}^b & \Omega_4{}^4 \end{pmatrix}, \quad \Omega_4{}^4 = -\Omega_a{}^a. \quad (30)$$

The 1-form  $h$  in (28) corresponds to the fiber direction of  $\mathbb{CP}^3 \times U(1)$ .

The fermionic sector of (20) decomposes as follows

$$\begin{aligned} F^{\alpha A'} O_{\alpha A'} &= \bar{F}^{\alpha A} \bar{O}_{\alpha A} + F_A{}^\alpha O_\alpha^A \\ &= \omega_A{}^\mu Q_\mu^A + \bar{\omega}^{\mu A} \bar{Q}_{\mu A} + \chi_{\mu A} S^{\mu A} + \bar{\chi}_\mu^A \bar{S}_A^\mu, \end{aligned} \quad (31)$$

where the generators have been split into the supersymmetry and superconformal ones (see Eq.(114)) accompanied by the corresponding splitting of the Cartan forms

$$\bar{F}^{\alpha A} = \sqrt{2} \begin{pmatrix} \bar{\omega}^{\mu A} \\ \bar{\chi}_\mu^A \end{pmatrix}, \quad F_A{}^\alpha = \sqrt{2} \begin{pmatrix} \omega_A{}^\mu \\ \chi_{\mu A} \end{pmatrix}. \quad (32)$$

The MC equations (5) in the novel basis acquire the form

$$\begin{aligned} d\omega^m &= 2G^m{}_n \wedge \omega^n + 2\Delta \wedge \omega^m - 2i\omega_A{}^\mu \sigma_{\mu\nu}^m \bar{\omega}^{\nu A}, \\ dc^m &= 2G^m{}_n \wedge c^n - 2\Delta \wedge c^m - 2i\chi_{\mu A} \tilde{\sigma}^{m\mu\nu} \bar{\chi}_\nu^A, \\ d\Delta &= \omega^m \wedge c_m + i\omega_A{}^\mu \wedge \bar{\chi}_\mu^A + i\bar{\omega}^{\mu A} \wedge \chi_{\mu A} \end{aligned} \quad (33)$$

for the Cartan forms from the  $AdS$  sector,

$$\begin{aligned}
d\Omega_a &= -i\tilde{\Omega}_a^b \wedge \Omega_b - i\tilde{\Omega}_b^b \wedge \Omega_a - i\varepsilon_{abc}\Omega_4^b \wedge \tilde{\Omega}^c + 2\varepsilon_{abc}\bar{\omega}^{\mu b} \wedge \bar{\chi}_\mu^c, \\
d\Omega^a &= -i\tilde{\Omega}^b \wedge \tilde{\Omega}_b^a - i\tilde{\Omega}^a \wedge \tilde{\Omega}_b^b + i\varepsilon^{abc}\Omega_b^4 \wedge \tilde{\Omega}_c - 2\varepsilon^{abc}\omega_b^\mu \wedge \chi_{\mu c}, \\
d\tilde{\Omega}_a &= -i\tilde{\Omega}_a^b \wedge \tilde{\Omega}_b + i\tilde{\Omega}_b^b \wedge \tilde{\Omega}_a + 2ih \wedge \tilde{\Omega}_a - i\varepsilon_{abc}\Omega_4^b \wedge \Omega^c - 2\omega_a^\mu \wedge \chi_{\mu 4} + 2\omega_4^\mu \wedge \chi_{\mu a}, \\
d\tilde{\Omega}^a &= -i\tilde{\Omega}^b \wedge \tilde{\Omega}_b^a + i\tilde{\Omega}^a \wedge \tilde{\Omega}_b^b - 2ih \wedge \tilde{\Omega}^a + i\varepsilon^{abc}\Omega_b^4 \wedge \Omega_c + 2\bar{\omega}^{\mu a} \wedge \bar{\chi}_{\mu 4} - 2\bar{\omega}^{\mu 4} \wedge \bar{\chi}_\mu^a, \\
d\tilde{\Omega}_a^a &= -i\tilde{\Omega}_a \wedge \Omega^a + \frac{i}{2}\tilde{\Omega}_a \wedge \tilde{\Omega}^a + \frac{i}{2}\Omega_a^4 \wedge \Omega_4^a - \omega_a^\mu \wedge \bar{\chi}_\mu^a + \bar{\omega}^{\mu a} \wedge \chi_{\mu a}, \\
dh &= \frac{i}{2}\tilde{\Omega}_a \wedge \tilde{\Omega}^a + \frac{i}{2}\Omega_4^a \wedge \Omega_a^4 - \omega_4^\mu \wedge \bar{\chi}_\mu^4 + \bar{\omega}^{\mu 4} \wedge \chi_{\mu 4}
\end{aligned} \tag{34}$$

for the Cartan forms from the  $so(8)$  sector and

$$\begin{aligned}
d\omega_a^\mu &= \Delta \wedge \omega_a^\mu + \frac{1}{2}G^{mn} \wedge \omega_a^\nu \sigma_{mn\nu}^\mu - \omega^m \wedge \tilde{\sigma}_m^{\mu\nu} \chi_{\nu a} - i\tilde{\Omega}_a^b \wedge \omega_b^\mu + i\tilde{\Omega}_b^b \wedge \omega_a^\mu \\
&\quad - i\Omega_a^4 \wedge \omega_4^\mu + i\tilde{\Omega}_a \wedge \bar{\omega}^{\mu 4} - i\varepsilon_{abc}\Omega^b \wedge \bar{\omega}^{\mu c}, \\
d\omega_4^\mu &= \Delta \wedge \omega_4^\mu + \frac{1}{2}G^{mn} \wedge \omega_4^\nu \sigma_{mn\nu}^\mu - \omega^m \wedge \tilde{\sigma}_m^{\mu\nu} \chi_{\nu 4} + 2ih \wedge \omega_4^\mu - i\Omega_4^a \wedge \omega_a^\mu - i\tilde{\Omega}_a \wedge \bar{\omega}^{\mu a}, \\
d\chi_{\mu a} &= -\Delta \wedge \chi_{\mu a} - \frac{1}{2}G^{mn} \wedge \sigma_{mn\mu}^\nu \chi_{\nu a} + c^m \wedge \sigma_{m\mu\nu} \omega_a^\nu - i\tilde{\Omega}_a^b \wedge \chi_{\mu b} + i\tilde{\Omega}_b^b \wedge \chi_{\mu a} \\
&\quad - i\Omega_a^4 \wedge \chi_{\mu 4} + i\tilde{\Omega}_a \wedge \bar{\chi}_\mu^4 - i\varepsilon_{abc}\Omega^b \wedge \bar{\chi}_\mu^c, \\
d\chi_{\mu 4} &= -\Delta \wedge \chi_{\mu 4} - \frac{1}{2}G^{mn} \wedge \sigma_{mn\mu}^\nu \chi_{\nu 4} + c^m \wedge \sigma_{m\mu\nu} \omega_4^\nu + 2ih \wedge \chi_{\mu 4} - i\Omega_4^a \wedge \chi_{\mu a} - i\tilde{\Omega}_a \wedge \bar{\chi}_\mu^a
\end{aligned} \tag{35}$$

and c.c. for the fermionic 1-forms.

In what follows we shall specialize to the  $OSp(4|8)/(SO(1,3) \times SO(7))$  representative in the form of the "dressed"  $OSp(4|6)/(SO(1,3) \times U(3))$  supercoset element

$$\mathcal{G} = \mathcal{G}_{OSp(4|6)/(SO(1,3) \times U(3))} e^{yH} e^{\theta_4^\mu Q_\mu^4 + \bar{\theta}^{\mu 4} \bar{Q}_{\mu 4}} e^{\eta_{\mu 4} S^{\mu 4} + \bar{\eta}_4^\mu \bar{S}_\mu^4}, \tag{36}$$

where

$$\mathcal{G}_{OSp(4|6)/(SO(1,3) \times U(3))} = e^{x^m P_m + \theta_a^\mu Q_\mu^a + \bar{\theta}^{\mu a} \bar{Q}_{\mu a}} e^{\eta_{\mu a} S^{\mu a} + \bar{\eta}_a^\mu \bar{S}_\mu^a} e^{z^a T_a + \bar{z}_a \bar{T}_a} e^{\varphi D} \tag{37}$$

is the  $OSp(4|6)/(SO(1,3) \times U(3))$  supercoset element considered in [21]. It is parametrized by the  $D=3$   $\mathcal{N}=6$  super-Poincare coordinates  $x^m, \theta_a^\mu, \bar{\theta}^{\mu a}$  supplemented by the coordinates  $\eta_{\mu a}, \bar{\eta}_\mu^a$  associated with the superconformal generators, the coordinates  $z^a, \bar{z}_a$  parametrizing  $\mathbb{CP}^3$  and the coordinate  $\varphi$  related to the radial direction of  $AdS_4$ . The expressions for the Cartan forms corresponding to the supercoset element (37) have been derived in [21]. The form of the supercoset element (36) is governed by the requirement of the absence of the vielbein dependence on the reduction direction coordinate  $y$  that can be satisfied by placing  $e^{yH}$  to the left from the factors corresponding to the broken supersymmetries of the background, whenever the differential acts from the right. The differentials of the factors corresponding to the broken supersymmetries give contributions to the  $sp(4)$  Cartan forms, the forms associated with the broken supersymmetries and the  $U(1)$  direction. After the commutation of  $\mathcal{G}_{OSp(4|6)/(SO(1,3) \times U(3))}^{-1} d\mathcal{G}_{OSp(4|6)/(SO(1,3) \times U(3))}$  with the factors corresponding to the broken supersymmetries the  $osp(4|6)$  Cartan forms  $\mathcal{C}$  become  $\mathcal{N}=4$  superfields  $\mathcal{C}(\theta_4, \bar{\theta}^4, \eta_4, \bar{\eta}^4)$ . There also appear the contributions proportional to the broken supersymmetries generators and  $\tilde{T}_a, \tilde{T}^a, V_a^4, V_4^a, H$  generators.

### 3 $AdS_4 \times \mathbb{CP}^3$ superstring in the light-cone gauge

The fermionic light-cone gauge condition we consider

$$\theta_A^2 = \bar{\theta}^{2A} = \eta_{1A} = \bar{\eta}_1^A = 0 \tag{38}$$

is characterized by setting to zero the coordinates associated with the generators  $Q_2^A$ ,  $\bar{Q}_{2A}$ ,  $S^{1A}$ ,  $\bar{S}_A^1$  that have negative charge w.r.t. the  $so(1,1)$  generator  $M^{+-} \equiv 2M^{02}$  from the Lorentz group acting on the Minkowski boundary of  $AdS_4$

$$[M^{+-}, Q_2^A] = -Q_2^A, \quad [M^{+-}, \bar{Q}_{2A}] = -\bar{Q}_{2A}, \quad [M^{+-}, S^{1A}] = -S^{1A}, \quad [M^{+-}, \bar{S}_A^1] = -\bar{S}_A^1. \quad (39)$$

Other fermionic generators have positive  $so(1,1)$  charge

$$[M^{+-}, Q_1^A] = Q_1^A, \quad [M^{+-}, \bar{Q}_{1A}] = \bar{Q}_{1A}, \quad [M^{+-}, S^{2A}] = S^{2A}, \quad [M^{+-}, \bar{S}_A^2] = \bar{S}_A^2. \quad (40)$$

Respective coordinates can be denoted as follows

$$\theta_A^1 \equiv \theta_A^- \equiv \theta_A, \quad \bar{\theta}^{1A} \equiv \bar{\theta}^{-A} \equiv \bar{\theta}^A, \quad \eta_A^1 \equiv \eta_A^- \equiv \eta_A, \quad \bar{\eta}^{1A} \equiv \bar{\eta}^{-A} \equiv \bar{\eta}^A \quad (41)$$

and become the physical degrees of freedom of the superstring in the gauge (38).

In the light-cone gauge the constituents of the  $AdS_4$  vielbein (23) reduce to

$$\begin{aligned} \omega^m &= e^{-2\varphi}(dx^m - id\theta_a\sigma^m\bar{\theta}^a + i\theta_a\sigma^m d\bar{\theta}^a) - id\theta_4\sigma^m\bar{\theta}^4 + i\theta_4\sigma^m d\bar{\theta}^4 + dy\omega_y^m, \quad \omega_y^m = 4\theta_4\sigma^m\bar{\theta}^4, \\ c^m &= -ie^{2\varphi}(d\eta_a\tilde{\sigma}^m\bar{\eta}^a - \eta_a\tilde{\sigma}^m d\bar{\eta}^a) - id\eta_4\tilde{\sigma}^m\bar{\eta}^4 + i\eta_4\tilde{\sigma}^m d\bar{\eta}^4 + dy c_y^m, \quad c_y^m = 4\eta_4\tilde{\sigma}^m\bar{\eta}^4, \\ \Delta &= d\varphi. \end{aligned} \quad (42)$$

The  $\mathbb{CP}^3 \times U(1)$  vielbein components acquire the form

$$\Omega_{4a}^8 = i(\Omega_a - 2e^{-\varphi}\hat{\eta}_a\eta_4 dx^+), \quad \Omega^{84a} = i(\Omega^a + 2e^{-\varphi}\hat{\eta}^a\bar{\eta}^4 dx^+), \quad (43)$$

$$\Omega^{87} = h + \tilde{\Omega}_a^{\phantom{a}a}, \quad (44)$$

where

$$h = dy - e^{-2\varphi}\eta_4\bar{\eta}^4 dx^+, \quad (45)$$

$$\hat{\eta}_a = T_a^{\phantom{a}b}\eta_b + T_{ab}\bar{\eta}^b, \quad \hat{\eta}^a = T^a_{\phantom{a}b}\bar{\eta}^b + T^{ab}\eta_b \quad (46)$$

and the matrix  $T_a^{\phantom{a}b}$  has been defined in Eq.(64) of [21]. The Cartan forms  $\Omega_a$ ,  $\Omega^a$  and  $\tilde{\Omega}_a^{\phantom{a}a}$  in the light-cone gauge become purely bosonic quantities equal to  $\Omega_{\mathbf{b}a}^{\phantom{a}4}$ ,  $\Omega_{\mathbf{b}4}^{\phantom{a}a}$  and  $\Omega_{\mathbf{b}a}^{\phantom{a}a}$  respectively given by Eq.(65) of [21]. Then the  $S^7$  part of the target-space metric acquires the Kaluza-Klein form

$$ds_{S^7}^2 = \Omega^{8I'}\Omega^{8I'} = \Omega^{87}\Omega^{87} + ds_{CP^3}^2 = (dy + a)^2 + ds_{CP^3}^2, \quad (47)$$

where

$$a(d) = \tilde{\Omega}_a^{\phantom{a}a} - e^{-2\varphi}\eta_4\bar{\eta}^4 dx^+ \quad (48)$$

and

$$ds_{CP^3}^2 = -\Omega_{4a}^8\Omega^{84a}. \quad (49)$$

In analogy with (16) in the Cartan forms (32) associated with the supersymmetry and superconformal generators it is also possible to single out the terms proportional to the reduction direction coordinate differential  $dy$

$$\omega_A^\mu = \tilde{\omega}_A^\mu + dy\omega_{yA}^\mu, \quad \bar{\omega}^{\mu A} = \tilde{\bar{\omega}}^{\mu A} + dy\bar{\omega}_{y\mu}^{\mu A}, \quad \chi_{\mu A} = \tilde{\chi}_{\mu A} + dy\chi_{y\mu A}, \quad \bar{\chi}_\mu^A = \tilde{\bar{\chi}}_\mu^A + dy\bar{\chi}_{y\mu}^A. \quad (50)$$

In the light-cone gauge the  $dy$ -independent components are given by

$$\tilde{\omega}_a^\mu = e^{-\varphi} \begin{pmatrix} \hat{d}\theta_a + dx^1\hat{\eta}_a \\ dx^+\hat{\eta}_a \end{pmatrix}, \quad \tilde{\bar{\omega}}^{\mu a} = e^{-\varphi} \begin{pmatrix} \hat{d}\bar{\theta}^a + dx^1\hat{\eta}^a \\ dx^+\hat{\eta}^a \end{pmatrix}; \quad (51)$$

$$\tilde{\chi}_{\mu a} = \begin{pmatrix} 0 \\ e^\varphi \hat{d}\eta_a \end{pmatrix}, \quad \bar{\tilde{\chi}}_\mu^a = \begin{pmatrix} 0 \\ e^\varphi \hat{d}\bar{\eta}^a \end{pmatrix}, \quad (52)$$

where  $\hat{d}\theta_a(\hat{d}\eta_a) = T_a{}^b d\theta_b(d\eta_b) + T_{ab} d\bar{\theta}^b(d\bar{\eta}^b)$ ,  $\hat{d}\bar{\theta}^a(\hat{d}\bar{\eta}^a) = T^a{}_b d\bar{\theta}^b(d\bar{\eta}^b) + T^{ab} d\theta_b(d\eta_b)$ , and

$$\tilde{\omega}_4^\mu = \begin{pmatrix} d\theta_4 + d\varphi\theta_4 + e^{-2\varphi} dx^1 \eta_4 \\ e^{-2\varphi} dx^+ \eta_4 \end{pmatrix}, \quad \bar{\tilde{\omega}}^{\mu 4} = \begin{pmatrix} d\bar{\theta}^4 + d\varphi\bar{\theta}^4 + e^{-2\varphi} dx^1 \bar{\eta}^4 \\ e^{-2\varphi} dx^+ \bar{\eta}^4 \end{pmatrix}; \quad (53)$$

$$\tilde{\chi}_{\mu 4} = \begin{pmatrix} 0 \\ d\eta_4 - d\varphi\eta_4 \end{pmatrix}, \quad \bar{\tilde{\chi}}_\mu^4 = \begin{pmatrix} 0 \\ d\bar{\eta}^4 - d\varphi\bar{\eta}^4 \end{pmatrix}. \quad (54)$$

The components of  $F_y^{\hat{\alpha}}$  acquire the form

$$\omega_{y4}^\mu = \begin{pmatrix} 2i\theta_4 \\ 0 \end{pmatrix}, \quad \bar{\omega}_y^{\mu 4} = \begin{pmatrix} -2i\bar{\theta}^4 \\ 0 \end{pmatrix}; \quad (55)$$

$$\chi_{y\mu 4} = \begin{pmatrix} 0 \\ 2i\eta_4 \end{pmatrix}, \quad \bar{\chi}_{y\mu}^4 = \begin{pmatrix} 0 \\ -2i\bar{\eta}^4 \end{pmatrix}. \quad (56)$$

For the  $OSp(4|8)/(SO(1,3) \times SO(7))$  supercoset element (36) the consequence of the absence of the vielbein dependence on the reduction direction coordinate  $y$  is that the  $AdS_4$  vielbein components (42) acquire the contributions proportional to  $dy$ . To remove them, i.e. to bring the bosonic components of the supervielbein to the Kaluza-Klein ansatz form (14), the local Lorentz rotation in the tangent space needs to be performed. Since the  $\mathbb{CP}^3$  vielbein components (43) do not contain the contributions proportional to  $dy$ , the necessary frame rotation  $L$  involves only tangent to the  $AdS_4$  directions and the one tangent to the  $U(1)$ -fiber direction on  $S^7$  labeled by "11" in the 11d context

$$\begin{pmatrix} E^{m'} \\ E^{11} \end{pmatrix} = L \begin{pmatrix} G^{0'm'} \\ \Omega^{87} \end{pmatrix}. \quad (57)$$

The entries of the matrix  $L$

$$L = \begin{pmatrix} L^{m'}{}_{n'} & L^{m'}{}_7 \\ L^7{}_{m'} & L^7{}_7 \end{pmatrix} \in SO(1,4) \quad (58)$$

in the light-cone gauge are given by

$$L^{m'}{}_{n'} = \delta_{n'}^{m'} - \frac{1}{2} G_y^{0'm'} G_{yn'}, \quad L^{m'}{}_7 = -G_y^{0'm'}, \quad L^7{}_{m'} = G_{ym'}^{0'}, \quad L^7{}_7 = 1, \quad (59)$$

where

$$G_y^{0'm'} = \left( \frac{1}{2} (\omega_y^m + c_y^m), 0 \right) = 2\Theta(1, 0, -1, 0), \quad \Theta = \theta_4 \bar{\theta}^4 + \eta_4 \bar{\eta}^4 \quad (60)$$

is the  $D = 1+3$  light-like vector. Corresponding Lorentz rotation acting on the supervielbein fermionic components is generated by the matrix

$$L^{\hat{\alpha}}{}_{\hat{\beta}} = \delta_{\hat{\beta}}^{\hat{\alpha}} - \frac{1}{2} G_{ym'}^{0'} \mathfrak{g}^{m'\hat{\alpha}}{}_{\hat{\gamma}} \mathfrak{g}^{11\hat{\gamma}}{}_{\hat{\beta}}. \quad (61)$$

As a result transformed bosonic components of the  $D = 11$  supervielbein in the light-cone basis equal

$$\begin{aligned} E^+ &= \frac{1}{2} e^{-2\varphi} dx^+, \quad E^- = \frac{1}{2} e^{-2\varphi} dx^- + \varpi - 2e^{-2\varphi} \Theta^2 dx^+ + 4\Theta(\tilde{\Omega}_a{}^a - e^{-2\varphi} \eta_4 \bar{\eta}^4 dx^+), \\ E^1 &= \frac{1}{2} e^{-2\varphi} dx^1, \quad E^3 = -d\varphi, \\ E^{11} &= dy + A(d), \quad A(d) = a(d) - e^{-2\varphi} \Theta dx^+ \end{aligned} \quad (62)$$



where  $x^\pm = x^2 \pm x^0$  and

$$\varpi = ie^{-2\varphi}(d\theta_a\bar{\theta}^a - \theta_a d\bar{\theta}^a) + i(d\theta_4\bar{\theta}^4 - \theta_4 d\bar{\theta}^4) + ie^{2\varphi}(d\eta_a\bar{\eta}^a - \eta_a d\bar{\eta}^a) + i(d\eta_4\bar{\eta}^4 - \eta_4 d\bar{\eta}^4). \quad (63)$$

Note that in the light-cone gauge the superfield  $\Phi = \sqrt{1 + G_y^{0'm'} G_{ym'}^{0'}}$  turns to unity for the chosen normalization. Correspondingly the transformed fermionic components  $(Lf)^{\hat{a}}$  of the supervielbein read

$$(L\tilde{\chi})_{\mu a} = \begin{pmatrix} 0 \\ e^\varphi \hat{d}\eta_a + 2ie^{-\varphi} \Theta \hat{\eta}_a dx^+ \end{pmatrix}, \quad (L\tilde{\chi})_\mu^a = \begin{pmatrix} 0 \\ e^\varphi \hat{d}\bar{\eta}^a - 2ie^{-\varphi} \Theta \hat{\eta}^a dx^+ \end{pmatrix} \quad (64)$$

and

$$(L\tilde{\chi})_{\mu 4} = \begin{pmatrix} 0 \\ d\eta_4 - d\varphi\eta_4 + 2ie^{-2\varphi} \Theta \eta_4 dx^+ \end{pmatrix}, \quad (L\tilde{\chi})_\mu^4 = \begin{pmatrix} 0 \\ d\bar{\eta}^4 - d\varphi\bar{\eta}^4 - 2ie^{-2\varphi} \Theta \bar{\eta}^4 dx^+ \end{pmatrix}. \quad (65)$$

Other fermionic vielbein components (51), (53), (55), (56) remain unaffected. Altogether they define the Kaluza-Klein form of the fermionic vielbein (15).

Then using (43) and (62) we find the expression for the induced world-sheet metric in the fermionic light-cone gauge (38)

$$g_{ij}^{(2)} = g_{ij}^{AdS} + g_{ij}^{CP}, \quad (66)$$

where

$$\begin{aligned} g_{ij}^{AdS} &= \frac{1}{2}(E_i^+ E_j^- + E_j^+ E_i^-) + E_i^1 E_j^1 + E_i^3 E_j^3 \\ &= \frac{1}{8}e^{-4\varphi}(\partial_i x^+ \partial_j x^- + \partial_j x^+ \partial_i x^-) + \frac{1}{4}e^{-4\varphi} \partial_i x^1 \partial_j x^1 + \partial_i \varphi \partial_j \varphi \\ &\quad + \frac{1}{4}e^{-2\varphi}[\partial_i x^+(\varpi_j + 4\Theta \tilde{\Omega}_{ja}^a) + \partial_j x^+(\varpi_i + 4\Theta \tilde{\Omega}_{ia}^a)] - 4e^{-4\varphi} \theta_4 \bar{\theta}^4 \eta_4 \bar{\eta}^4 \partial_i x^+ \partial_j x^+ \end{aligned} \quad (67)$$

and

$$\begin{aligned} g_{ij}^{CP} &= -\frac{1}{2}(\Omega_i^{84a} \Omega_j^{84a} + \Omega_j^{84a} \Omega_i^{84a}) \\ &= \frac{1}{2}(\Omega_{ia} \Omega_j^a + \Omega_{ja} \Omega_i^a) + e^{-\varphi}[\partial_i x^+(\Omega_{ja} \hat{\eta}^a \bar{\eta}^4 - \Omega_j^a \hat{\eta}_a \eta_4) + \partial_j x^+(\Omega_{ia} \hat{\eta}^a \bar{\eta}^4 - \Omega_i^a \hat{\eta}_a \eta_4)] \\ &\quad + 4e^{-2\varphi} \hat{\eta}_a \hat{\eta}^a \eta_4 \bar{\eta}^4 \partial_i x^+ \partial_j x^+. \end{aligned} \quad (68)$$

The superstring WZ term is determined by the integral of the NS-NS 3-form (19)

$$H_{(3)} = \frac{i}{4}(E^{\hat{\alpha}} \mathbf{g}^{\hat{m}'\hat{n}'} \hat{\alpha}^{\hat{\beta}} \chi_{\hat{\beta}} \wedge E_{\hat{m}'} \wedge E_{\hat{n}'} + E^{\hat{\alpha}} \mathbf{g}^{\hat{m}'11} \hat{\alpha}^{\hat{\beta}} \wedge E_{\hat{\beta}} \wedge E_{\hat{m}'} - \varepsilon_{m'n'k'l'} E^{m'} \wedge E^{n'} \wedge E^{k'} L^{l'} \wedge \gamma_7) \quad (69)$$

that in the fermionic light-cone gauge (38) can be presented as the total differential of the 2-form

$$\begin{aligned} B_{(2)} &= \frac{1}{2}e^{-4\varphi}(\theta_4 \bar{\theta}^4 + \eta_4 \bar{\eta}^4) dx^1 \wedge dx^+ \\ &\quad + \frac{1}{4}e^{-2\varphi}(d\theta_4 \bar{\eta}^4 - d\eta_4 \bar{\theta}^4 + \eta_4 d\bar{\theta}^4 - \theta_4 d\bar{\eta}^4) \wedge dx^+ \\ &\quad + ie^{-2\varphi}(\theta_4 \bar{\eta}^4 - \eta_4 \bar{\theta}^4) dx^+ \wedge \tilde{\Omega}_a^a + ie^{-\varphi} \hat{\eta}_a \theta_4 dx^+ \wedge \Omega^a + ie^{-\varphi} \hat{\eta}^a \bar{\theta}^4 dx^+ \wedge \Omega_a \\ &\quad + e^{-2\varphi} \hat{\eta}_a \hat{\eta}^a dx^1 \wedge dx^+ + \frac{1}{2}e^{-2\varphi}(\hat{\eta}_a \hat{d}\bar{\theta}^a + \hat{d}\theta_a \hat{\eta}^a) \wedge dx^+. \end{aligned} \quad (70)$$

So the Polyakov representation of the  $AdS_4 \times \mathbb{CP}^3$  superstring action in the fermionic light-cone gauge (38) becomes

$$S_{l.c.} = -\frac{1}{2} \int_{\Sigma} d\tau d\sigma \sqrt{-\gamma} \gamma^{ij} (g_{ij}^{AdS} + g_{ij}^{CP}) \pm \int_{\Sigma} d\tau d\sigma B_{(2)}, \quad (71)$$

where  $\gamma_{ij}$  is the auxiliary world-sheet metric. The bosonic light-cone gauge condition can be fixed in various ways, in particular, within the Hamiltonian approach [20], [12].

## 4 Conclusion

We have obtained the  $AdS_4 \times \mathbb{CP}^3$  superstring action in the light-cone gauge, in which both light-like directions lie on the Minkowski boundary of the  $AdS_4$  part of the background. In deriving it we employed the double dimension reduction of the supermembrane action on the  $AdS_4 \times S^7$  background constructed in [11] using the supercoset approach. To perform the dimensional reduction it is necessary to change the generator basis in the  $so(8)$  sector of the  $osp(4|8)$  isometry superalgebra of  $AdS_4 \times S^7$  superbackground to exhibit the Hopf fibration realization of the 7-sphere. In the  $sp(4)$  sector the generators have been transformed to the  $3d$  conformal basis. As a result the fermionic generators and coordinates naturally split into corresponding to the (un)broken Poincare and conformal supersymmetries. The kinetic term of the resultant action includes the contributions up to the 4th order in the space-time fermions and the WZ term is of the 2nd order similarly to the  $AdS_5 \times S^5$  superstring case [20]. There can be taken simplifying limits of the action analogous to those considered for the  $AdS_5 \times S^5$  superstring (for review see [12]). It can also be used to study the string states with zero quantum numbers from the  $\mathbb{CP}^3$  sector within the semiclassical approximation [27], [28], whose application for the  $AdS_4 \times \mathbb{CP}^3$  superstring has been initiated in [29].

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## A Notation and spinor properties

We use the following notation

- for the vector indices
  - in  $D = 1 + 10$  dimensions  $\hat{m}, \hat{n} = 0, 1, \dots, 9, 11$ ;
  - in  $D = 1 + 9$  dimensions  $\hat{m}', \hat{n}' = 0, 1, \dots, 9$ ;
  - in  $D = 8$  dimensions  $\underline{I}, \underline{J}, \underline{K} = 1, \dots, 8$ ;
  - in  $D = 7$  dimensions  $I', J', K' = 1, \dots, 7$ ;
  - in  $D = 6$  dimensions  $I, J, K = 1, \dots, 6$ ;
  - in  $D = 2 + 3$  dimensions  $\underline{m}, \underline{n} = 0', 0, \dots, 3$ ;
  - in  $D = 1 + 3$  dimensions  $m', n' = 0, \dots, 3$ ;
  - in  $D = 1 + 2$  dimensions  $m, n = 0, 1, 2$ ;
- for the membrane coordinate indices  $\underline{i}, \underline{j} = \tau, \sigma, y$ ;
- for the string coordinate indices  $i, j = \tau, \sigma$ ;
- for the spinor indices
  - in  $D = 10, 11$  dimensions  $\hat{\alpha}, \hat{\beta}, \hat{\gamma} = 1, \dots, 32$ ;
  - in  $D = 7, 8$  dimensions  $A', B', C' = 1, \dots, 8$ ;
  - in  $D = 6$  dimensions  $A, B, C = 1, \dots, 4$ ;
  - of the  $SU(3)$  fundamental representation  $a, b, c = 1, 2, 3$ ;

- in  $D = 4, 5$  dimensions  $\alpha, \beta, \gamma = 1, \dots, 4$ ;
- in  $D = 3$  dimensions  $\mu, \nu, \lambda = 1, 2$ .

The definition of the  $D = 2 + 3$   $\gamma$ -matrices  $\gamma_{\alpha\beta}^m$  is that adopted in [21]. The  $D = 1 + 3$   $\gamma$ -matrices are defined as

$$\Gamma^{m'}_{\alpha}{}^{\beta} = \gamma^{m'}_{\alpha}{}^{\beta}, \quad \Gamma^5_{\alpha}{}^{\beta} = (\Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3)_{\alpha}{}^{\beta} : \quad \Gamma^5_{\alpha} \gamma \Gamma^5_{\gamma}{}^{\beta} = -\delta_{\alpha}^{\beta} \quad (72)$$

and satisfy the Clifford algebra relations

$$\Gamma^{m'}_{\alpha} \gamma \Gamma^{n'}_{\gamma}{}^{\beta} + \Gamma^{n'}_{\alpha} \gamma \Gamma^{m'}_{\gamma}{}^{\beta} = 2\eta^{m'n'} \delta_{\alpha}^{\beta}. \quad (73)$$

The antisymmetric  $D = 1 + 3$  charge conjugation matrices are

$$C'_{\alpha\beta} = \gamma_{\alpha\beta}^{0'}, \quad C'^{\alpha\beta} = \tilde{\gamma}^{0'\alpha\beta}. \quad (74)$$

Such definition implies the relation

$$C_{\alpha\beta} = \Gamma^5_{\alpha} \gamma C'_{\gamma\beta}, \quad (75)$$

where  $C_{\alpha\beta}$  is the  $Sp(4)$  antisymmetric metric. It follows that  $\Gamma^{m'}_{\alpha\beta} = -\Gamma^{m'}_{\alpha} \gamma C'_{\gamma\beta}$  and the  $so(1, 3)$  generators

$$\Gamma^{m'n'}_{\alpha\beta} = -\frac{1}{2}(\Gamma^{m'}_{\alpha} \gamma \Gamma^{n'}_{\gamma}{}^{\delta} - \Gamma^{n'}_{\alpha} \gamma \Gamma^{m'}_{\gamma}{}^{\delta}) C'_{\delta\beta} \quad (76)$$

are symmetric in the spinor indices, while other antisymmetrized products of  $4d$   $\gamma$ -matrices are antisymmetric.

In terms of  $D = 1 + 2$  real symmetric  $\gamma$ -matrices  $\sigma_{\mu\nu}^m$ ,  $\tilde{\sigma}^{m\mu\nu}$  and antisymmetric metric tensors  $\varepsilon_{\mu\nu}$ ,  $\varepsilon^{\mu\nu}$  introduced in [21] the  $4d$   $\gamma$ -matrices and  $so(1, 3)$  generators are expressed as

$$\Gamma^m_{\alpha}{}^{\beta} = -\begin{pmatrix} \sigma^m_{\mu}{}^{\nu} & 0 \\ 0 & \sigma^m_{\mu}{}^{\nu} \end{pmatrix}, \quad \Gamma^3_{\alpha}{}^{\beta} = \begin{pmatrix} 0 & \varepsilon_{\mu\nu} \\ \varepsilon^{\mu\nu} & 0 \end{pmatrix}, \quad \Gamma^5_{\alpha}{}^{\beta} = \begin{pmatrix} 0 & -\varepsilon_{\mu\nu} \\ \varepsilon^{\mu\nu} & 0 \end{pmatrix} \quad (77)$$

and

$$\Gamma^{mn}_{\alpha}{}^{\beta} = \begin{pmatrix} -\sigma^{mn}_{\mu}{}^{\nu} & 0 \\ 0 & \sigma^{mn}_{\mu}{}^{\nu} \end{pmatrix}, \quad \Gamma^{3m}_{\alpha}{}^{\beta} = -\begin{pmatrix} 0 & \sigma^m_{\mu\nu} \\ \tilde{\sigma}^{m\mu\nu} & 0 \end{pmatrix}, \quad (78)$$

where  $\sigma^{mn}_{\mu}{}^{\nu} = \frac{1}{2}(\sigma^m_{\mu\lambda} \tilde{\sigma}^{n\lambda\nu} - (m \leftrightarrow n))$ .

The  $D = 7$   $\gamma$ -matrices satisfy the following Clifford algebra relations

$$\gamma^{I'}_{A'} C' \gamma^{J'}_{C'} B' + \gamma^{J'}_{A'} C' \gamma^{I'}_{C'} B' = -2\delta^{I'J'} \delta_{A'}^{B'} \quad (79)$$

and are antisymmetric w.r.t. symmetric charge conjugation matrix  $C_{A'B'}$  and its inverse  $C^{A'B'}$

$$\gamma^{I'}_{A'B'} = \gamma^{I'}_{A'} C' C_{B'} : \quad \gamma^{I'}_{A'B'} = -\gamma^{I'}_{B'A'}. \quad (80)$$

The  $so(7)$  generators are defined as

$$\gamma^{I'J'}_{A'} B' = \frac{1}{2}(\gamma^{I'}_{A'} C' \gamma^{J'}_{C'} B' - \gamma^{J'}_{A'} C' \gamma^{I'}_{C'} B') \quad (81)$$

and are also antisymmetric in the spinor indices.

From the  $6d$  perspective the  $D = 7$  Majorana spinor is composed of the 4-component  $SU(4)$  spinor and its conjugate

$$\psi_{A'} = \begin{pmatrix} \bar{\psi}_A \\ \psi^A \end{pmatrix}, \quad \bar{\psi}^{A'} = \begin{pmatrix} \psi^A \\ \bar{\psi}_A \end{pmatrix}. \quad (82)$$

Accordingly  $D = 7$  charge conjugation and  $\gamma$ -matrices have the following representation in terms of the  $D = 6$  chiral  $\gamma$ -matrices  $\rho_{AB}^I$ ,  $\tilde{\rho}^{IAB}$  antisymmetric in the spinor indices and  $4 \times 4$  unit matrix

$$C_{A'B'} = - \begin{pmatrix} 0 & \delta_A^B \\ \delta_B^A & 0 \end{pmatrix}, \quad C^{A'B'} = - \begin{pmatrix} 0 & \delta_B^A \\ \delta_A^B & 0 \end{pmatrix} \quad (83)$$

and

$$\gamma_{A'B'}^I = \begin{pmatrix} \rho_{AB}^I & 0 \\ 0 & -\tilde{\rho}^{IAB} \end{pmatrix}, \quad \gamma_{A'B'}^7 = i \begin{pmatrix} 0 & -\delta_A^B \\ \delta_B^A & 0 \end{pmatrix}. \quad (84)$$

Then the  $so(7)$  generators (81) acquire the form

$$\gamma^{IJ}{}_{A'B'} = \begin{pmatrix} -\rho^{IJ}{}_{AB} & 0 \\ 0 & \rho^{IJ}{}_{BA} \end{pmatrix}, \quad \gamma^{7I}{}_{A'B'} = -i \begin{pmatrix} 0 & \rho_{AB}^I \\ \tilde{\rho}^{IAB} & 0 \end{pmatrix}. \quad (85)$$

The  $D = 1 + 10$   $\gamma$ -matrices  $\mathbf{g}^{\hat{m}}{}_{\hat{\alpha}}{}^{\hat{\beta}}$  that are symmetric w.r.t. antisymmetric charge conjugation matrix  $\mathbf{c}_{\hat{\alpha}\hat{\beta}}$  and satisfy the  $D = 11$  Clifford algebra relations

$$\mathbf{g}^{\hat{m}}{}_{\hat{\alpha}}{}^{\hat{\gamma}} \mathbf{g}^{\hat{n}}{}_{\hat{\gamma}}{}^{\hat{\beta}} + \mathbf{g}^{\hat{n}}{}_{\hat{\alpha}}{}^{\hat{\gamma}} \mathbf{g}^{\hat{m}}{}_{\hat{\gamma}}{}^{\hat{\beta}} = 2\eta^{\hat{m}\hat{n}} \delta_{\hat{\alpha}}^{\hat{\beta}} \quad (86)$$

have the following realization in terms of the  $4d$  and  $7d$  matrices

$$\mathbf{g}^{m'}{}_{\hat{\alpha}}{}^{\hat{\beta}} = \delta_{A'}^{B'} \Gamma^{m'}{}_{\alpha}{}^{\beta}, \quad \mathbf{g}^I{}_{\hat{\alpha}}{}^{\hat{\beta}} = -\Gamma^5{}_{\alpha}{}^{\beta} \gamma^I{}_{A'}{}^{B'}, \quad \mathbf{g}^{11}{}_{\hat{\alpha}}{}^{\hat{\beta}} = -\Gamma^5{}_{\alpha}{}^{\beta} \gamma^7{}_{A'}{}^{B'}. \quad (87)$$

The charge conjugation matrix  $\mathbf{c}_{\hat{\alpha}\hat{\beta}}$  is defined as the direct product of the  $4d$  and  $7d$  charge conjugation matrices

$$\mathbf{c}_{\hat{\alpha}\hat{\beta}} = C'_{\alpha\beta} C_{A'B'}. \quad (88)$$

Then the  $so(1, 10)$  generators

$$\mathbf{g}^{\hat{m}\hat{n}}{}_{\hat{\alpha}}{}^{\hat{\beta}} = \frac{1}{2} (\mathbf{g}^{\hat{m}}{}_{\hat{\alpha}}{}^{\hat{\gamma}} \mathbf{g}^{\hat{n}}{}_{\hat{\gamma}}{}^{\hat{\beta}} - \mathbf{g}^{\hat{n}}{}_{\hat{\alpha}}{}^{\hat{\gamma}} \mathbf{g}^{\hat{m}}{}_{\hat{\gamma}}{}^{\hat{\beta}}) \quad (89)$$

split into the  $so(1, 3)$  generators  $\mathbf{g}^{m'n'}{}_{\hat{\alpha}}{}^{\hat{\beta}} = \delta_{A'}^{B'} \Gamma^{m'n'}{}_{\alpha}{}^{\beta}$  and  $so(7)$  generators  $\mathbf{g}^{I'J'}{}_{\hat{\alpha}}{}^{\hat{\beta}} = -\delta_{\alpha}^{\beta} \gamma^{I'J'}{}_{A'}{}^{B'}$ . Such realization of the  $D = 1 + 10$   $\gamma$ -matrices allows to bring the contribution of the  $D = 1 + 10$  translation generators  $P_{\hat{m}} = (M_{0'm'}, V^{8I'})$  to the anticommutator (110) of the  $osp(4|8)$  odd generators to the  $(1 + 10)$ -covariant form as

$$\{O_{\alpha A'}, O_{\beta B'}\} : \quad \frac{i}{2} C_{A'B'} \Gamma^{m'}{}_{\alpha}{}^{\gamma} C'_{\gamma\beta} M_{0'm'} - \frac{i}{2} \Gamma^5{}_{\alpha}{}^{\gamma} C'_{\gamma\beta} \gamma^{I'}{}_{A'B'} V^{8I'} = \frac{i}{2} \mathbf{g}^{\hat{m}}{}_{\hat{\alpha}}{}^{\hat{\gamma}} \mathbf{c}_{\hat{\gamma}\hat{\beta}} P_{\hat{m}}. \quad (90)$$

## B $osp(4|8)$ superalgebra

In analogy with the  $osp(4|6)$  superalgebra the (anti)commutation relations of  $osp(4|8)$  can be arranged into 5 sets

$$[O_{\alpha\beta}, O_{\gamma\delta}] = i(C_{\alpha\gamma} O_{\beta\delta} + C_{\alpha\delta} O_{\beta\gamma} + C_{\beta\gamma} O_{\alpha\delta} + C_{\beta\delta} O_{\alpha\gamma}), \quad (91)$$

$$[O_{A'B'}, O_{C'D'}] = \delta_{A'C'} O_{B'D'} - \delta_{A'D'} O_{B'C'} + \delta_{B'D'} O_{A'C'} - \delta_{B'C'} O_{A'D'}, \quad (92)$$

$$\{O_{\alpha A'}, O_{\beta B'}\} = -C_{A'B'} O_{\alpha\beta} + i C_{\alpha\beta} O_{A'B'}, \quad (93)$$

$$[O_{A'B'}, O_{\alpha C'}] = C_{A'C'} O_{\alpha B'} - C_{B'C'} O_{\alpha A'}, \quad (94)$$

$$[O_{\alpha\beta}, O_{\gamma A'}] = i(C_{\alpha\gamma} O_{\beta A'} + C_{\beta\gamma} O_{\alpha A'}). \quad (95)$$

Note that in the realization of the  $osp(4|8)$  superalgebra relevant for the membrane on  $AdS_4 \times S^7$  fermionic generators  $O_{\alpha A'}$  carry the  $8d$  chiral spinor index [11].

Eq. (91) defines the  $sp(4) \sim so(2, 3)$  algebra commutation relations that can be cast into the form of  $ads_4$  or  $conf_3$  algebra relations [21]. The difference here is in the normalization of the  $AdS_4$  translation generators  $M_{0'm'}$  that implies

$$O_{\alpha\beta} = -\frac{i}{2}\gamma_{\alpha\beta}^{\underline{mn}} M_{\underline{mn}} = -\frac{i}{2}(\Gamma^{m'}_{\alpha} \gamma C'_{\gamma\beta} M_{0'm'} + \Gamma^{m'n'}_{\alpha} \gamma \Gamma^5_{\gamma} C'_{\delta\beta} M_{m'n'}). \quad (96)$$

With the definition of the  $3d$  conformal generators

$$P_m = -M_{3m} + \frac{1}{2}M_{0'm'}, \quad K_m = M_{3m} + \frac{1}{2}M_{0'm'}, \quad D = -M_{0'3} \quad (97)$$

the commutation relations of the  $3d$  conformal algebra remain the same as in Eq.(B14) of [21].

The  $so(8)$  commutation relations (92) by the transformation

$$O_{A'B'} = -\frac{1}{4}\gamma_{A'B'}^{\underline{IJ}} V^{\underline{IJ}} = -\frac{1}{2}\gamma_{A'B'}^{I'} V^{8I'} + \frac{1}{4}\gamma_{A'B'}^{I'J'} V^{I'J'} \quad (98)$$

are converted into the vector form

$$[V^{\underline{IJ}}, V^{\underline{KL}}] = \delta^{\underline{IL}} V^{\underline{JK}} - \delta^{\underline{IK}} V^{\underline{JL}} + \delta^{\underline{JK}} V^{\underline{IL}} - \delta^{\underline{JL}} V^{\underline{IK}} \quad (99)$$

that generalizes the commutation relations of the  $so(6)$  subalgebra of  $osp(4|6)$  [21].

In terms of the  $D = 7$  generators the commutator (99) decomposes as

$$\begin{aligned} [V^{8I'}, V^{8J'}] &= -V^{I'J'}, \quad [V^{I'J'}, V^{8K'}] = \delta^{J'K'} V^{8I'} - \delta^{I'K'} V^{8J'}, \\ [V^{I'J'}, V^{K'L'}] &= \delta^{I'L'} V^{J'K'} - \delta^{J'K'} V^{I'L'} + \delta^{J'K'} V^{I'L'} - \delta^{J'L'} V^{I'K'} \end{aligned} \quad (100)$$

or further in terms of the  $D = 6$  generators

$$\begin{aligned} [V^{78}, V^{7I}] &= -V^{8I}, \quad [V^{78}, V^{8I}] = V^{7I}, \\ [V^{7I}, V^{7J}] &= [V^{8I}, V^{8J}] = -V^{IJ}, \quad [V^{7I}, V^{8J}] = -\delta^{IJ} V^{78}, \\ [V^{IJ}, V^{7(8)K}] &= \delta^{JK} V^{7(8)I} - \delta^{IK} V^{7(8)J}, \\ [V^{IJ}, V^{KL}] &= \delta^{IL} V^{JK} - \delta^{IK} V^{JL} + \delta^{JK} V^{IL} - \delta^{JL} V^{IK}, \end{aligned} \quad (101)$$

where the last commutator corresponds to the  $so(6)$  algebra.

The  $so(8)$  generators decompose into the following  $SU(3)$  irreducible parts

$$V^{78}, \quad V^7_{4a}, \quad V^8_{4a}, \quad V_a^4, \quad V_a^b - \frac{1}{3}\delta_a^b V_c^c, \quad V_a^a \quad (102)$$

and c.c., where we adopted the definition of the  $su(4)$  generators

$$V_A^B = \frac{i}{4}\rho^{IJ}{}_A{}^B V^{IJ} = \begin{pmatrix} V_a^b & V_a^4 \\ V_4^b & V_4^4 \end{pmatrix}, \quad V_4^4 = -V_a^a \quad (103)$$

and  $V^{7(8)}_{4a} = V^{7(8)I} \rho_{4a}^I$ .

The representation of the 7-sphere isometries in the form suitable for exhibiting its Hopf fibration structure requires the transformation of the  $so(8)$  generators (102). The novel generators

$$T_a = \frac{1}{2}(V^7_{4a} - iV^8_{4a}), \quad T^a = -\frac{1}{2}(V^{74a} + iV^{84a}) \quad (104)$$

are identified with the coset  $su(4)/u(3)$ . Their form is dictated by the generation of the  $su(4)$  algebra commutation relations and commutativity with the  $u(1)$  fiber generator

$$H = 2V_a^a - V^{78}. \quad (105)$$

The generators

$$\tilde{V}_a^b = V_a^b - \frac{1}{2}\delta_a^b V_c^c - \frac{1}{4}\delta_a^b V^{78} \quad (106)$$

are identified with the  $u(3)$  subgroup of  $su(4)$  and  $\tilde{V}_a^a = -\frac{1}{2}V_a^a - \frac{3}{4}V^{78}$  with the  $u(1)$  subgroup of  $u(3)$ . 15 Generators (104), (106) indeed provide the realization of the  $su(4)$  algebra

$$\begin{aligned} [T_a, T^b] &= i(\tilde{V}_a^b + \delta_a^b \tilde{V}_c^c), \quad [T_a, \tilde{V}_b^c] = -i\delta_a^c T_b, \quad [T^a, \tilde{V}_b^c] = i\delta_b^a T^c, \\ [\tilde{V}_a^b, \tilde{V}_c^d] &= i(\delta_c^b \tilde{V}_a^d - \delta_a^d \tilde{V}_c^b) \end{aligned} \quad (107)$$

with other commutation relations vanishing. Remaining 12 generators from the  $so(8)/(su(4) \times u(1))$  coset

$$\tilde{T}_a = -\frac{1}{2}(V_{4a}^7 + iV_{4a}^8), \quad \tilde{T}^a = \frac{1}{2}(V^{74a} - iV^{84a}), \quad V_a^4, \quad V_4^a \quad (108)$$

satisfy the following nonzero commutation relations between themselves and with the above introduced  $su(4)$  generators

$$\begin{aligned} [T_a, \tilde{T}_b] &= -i\varepsilon_{abc} V_4^c, \quad [T^a, \tilde{T}^b] = i\varepsilon^{abc} V_c^4, \\ [\tilde{T}_a, H] &= 2i\tilde{T}_a, \quad [\tilde{T}^a, H] = -2i\tilde{T}^a, \\ [\tilde{T}_a, \tilde{T}^b] &= i(\tilde{V}_a^b - \delta_a^b(\tilde{V}_c^c + \frac{H}{2})), \quad [\tilde{T}_a, \tilde{V}_b^c] = \frac{i}{2}\delta_b^c \tilde{T}_a - i\delta_a^c \tilde{T}_b, \quad [\tilde{T}^a, \tilde{V}_b^c] = -\frac{i}{2}\delta_b^c \tilde{T}^a + i\delta_b^a \tilde{T}^c, \\ [V_4^a, V_b^4] &= -i(\tilde{V}_b^a + \delta_b^a(\frac{H}{2} - \tilde{V}_c^c)), \quad [V_4^a, \tilde{V}_b^c] = -\frac{i}{2}\delta_b^c V_4^a + i\delta_b^a V_4^c, \quad [V_a^4, \tilde{V}_b^c] = \frac{i}{2}\delta_b^c V_a^4 - i\delta_a^c V_b^4, \\ [V_a^4, H] &= -2iV_a^4, \quad [V_4^a, H] = 2iV_4^a, \\ [V_a^4, T_b] &= -i\varepsilon_{abc} \tilde{T}^c, \quad [V_a^4, \tilde{T}_b] = -i\varepsilon_{abc} T^c, \\ [V_4^a, T^b] &= i\varepsilon^{abc} \tilde{T}_c, \quad [V_4^a, \tilde{T}^b] = i\varepsilon^{abc} T_c. \end{aligned} \quad (109)$$

The anticommutation relations (93) allow the following  $SO(1,3) \times SO(7)$  covariant representation

$$\begin{aligned} \{O_{\alpha A'}, O_{\beta B'}\} &= \frac{i}{2}C_{A'B'}(\Gamma^{m'}_{\alpha}{}^{\gamma}C'_{\gamma\beta}M_{0'm'} + \Gamma^{m'n'}_{\alpha}{}^{\gamma}\Gamma^5_{\gamma}{}^{\delta}C'_{\delta\beta}M_{m'n'}) \\ &- \frac{i}{4}\Gamma^5_{\alpha}{}^{\gamma}C'_{\gamma\beta}(2\gamma_{A'B'}^{I'}V^{8I'} - \gamma_{A'B'}^{I'J'}V^{I'J'}). \end{aligned} \quad (110)$$

The generators  $O_{\alpha A'}$  split into the pair of  $SU(4)$  chiral spinors

$$O_{\alpha A'} = \begin{pmatrix} \bar{O}_{\alpha A} \\ O_{\alpha}^A \end{pmatrix} \quad (111)$$

and the matrix of the  $so(8)$  generators has the following  $su(4)$  block structure

$$2\gamma_{A'B'}^{I'}V^{8I'} - \gamma_{A'B'}^{I'J'}V^{I'J'} = 2 \begin{pmatrix} V_{AB}^8 - iV_{AB}^7 & i\delta_A^B V^{78} + 2iV_A^B \\ -i\delta_B^A V^{78} - 2iV_B^A & -V^{8AB} - iV^{7AB} \end{pmatrix}. \quad (112)$$

Therefore the anticommutator (110) decomposes as follows

$$\begin{aligned} \{O_{\alpha}^A, O_{\beta}^B\} &= \frac{i}{2}\Gamma^5_{\alpha}{}^{\gamma}C'_{\gamma\beta}(V^{8AB} + iV^{7AB}), \quad \{\bar{O}_{\alpha A}, \bar{O}_{\beta B}\} = -\frac{i}{2}\Gamma^5_{\alpha}{}^{\gamma}C'_{\gamma\beta}(V_{AB}^8 - iV_{AB}^7), \\ \{O_{\alpha}^A, \bar{O}_{\beta B}\} &= -\frac{i}{2}\delta_B^A(\Gamma^{m'}_{\alpha}{}^{\gamma}C'_{\gamma\beta}M_{0'm'} + \Gamma^{m'n'}_{\alpha}{}^{\gamma}\Gamma^5_{\gamma}{}^{\delta}C'_{\delta\beta}M_{m'n'}) + \frac{i}{2}\Gamma^5_{\alpha}{}^{\gamma}C'_{\gamma\beta}(i\delta_B^A V^{78} + 2iV_B^A). \end{aligned} \quad (113)$$

Further decomposing the fermionic generators into the supersymmetry and superconformal ones

$$O_\alpha^A = \frac{1}{\sqrt{2}} \begin{pmatrix} Q_\mu^A \\ S^{\mu A} \end{pmatrix}, \quad \bar{O}_{\alpha A} = \frac{1}{\sqrt{2}} \begin{pmatrix} \bar{Q}_{\mu A} \\ \bar{S}_A^\mu \end{pmatrix}, \quad (114)$$

using the realization of  $\Gamma$ -matrices in terms of  $D = 1 + 2$  ones (77), (78), the definitions of the  $3d$  conformal group generators (97) and the  $so(8)$  generators (104)-(106), (108) we arrive at the set of anticommutators of the  $D = 3$   $N = 8$  superconformal algebra

$$\begin{aligned} \{Q_\mu^A, \bar{Q}_{\nu B}\} &= 2i\delta_B^A \sigma_{\mu\nu}^m P_m, \quad \{S^{\mu A}, \bar{S}_B^\nu\} = 2i\delta_B^A \tilde{\sigma}^{m\mu\nu} K_m \\ \{Q_\mu^a, \bar{S}_b^\nu\} &= -i\delta_\mu^\nu \delta_b^a D + i\delta_b^a \sigma^{mn}{}_\mu{}^\nu M_{mn} - 2\delta_\mu^\nu (\tilde{V}_b^a - \delta_b^a \tilde{V}_c^c), \\ \{\bar{Q}_{\mu a}, S^{\nu b}\} &= -i\delta_\mu^\nu \delta_a^b D + i\delta_a^b \sigma^{mn}{}_\mu{}^\nu M_{mn} + 2\delta_\mu^\nu (\tilde{V}_a^b - \delta_a^b \tilde{V}_c^c), \\ \{Q_\mu^4, \bar{S}_4^\nu\} &= -i\delta_\mu^\nu D + i\sigma^{mn}{}_\mu{}^\nu M_{mn} + \delta_\mu^\nu H, \quad \{\bar{Q}_{\mu 4}, S^{\nu 4}\} = -i\delta_\mu^\nu D + i\sigma^{mn}{}_\mu{}^\nu M_{mn} - \delta_\mu^\nu H, \\ \{Q_\mu^4, \bar{S}_b^\nu\} &= -2\delta_\mu^\nu V_b^4, \quad \{\bar{Q}_{\mu 4}, S^{\nu b}\} = 2\delta_\mu^\nu V_4^b, \\ \{Q_\mu^a, \bar{S}_4^\nu\} &= -2\delta_\mu^\nu V_4^a, \quad \{\bar{Q}_{\mu a}, S^{\nu 4}\} = 2\delta_\mu^\nu V_a^4, \\ \{Q_\mu^a, S^{\nu b}\} &= 2\delta_\mu^\nu \varepsilon^{abc} T_c, \quad \{\bar{Q}_{\mu a}, \bar{S}_b^\nu\} = -2\delta_\mu^\nu \varepsilon_{abc} T^c, \\ \{Q_\mu^a, S^{\nu 4}\} &= 2\delta_\mu^\nu \tilde{T}^a, \quad \{\bar{Q}_{\mu a}, \bar{S}_4^\nu\} = -2\delta_\mu^\nu \tilde{T}_a, \\ \{Q_\mu^4, S^{\nu a}\} &= -2\delta_\mu^\nu \tilde{T}^a, \quad \{\bar{Q}_{\mu 4}, \bar{S}_a^\nu\} = 2\delta_\mu^\nu \tilde{T}_a. \end{aligned} \quad (115)$$

Note that the anticommutators of the  $D = 3$   $N = 6$  superconformal algebra are the same as in [21] modulo renaming the  $su(4)$  generators.

The commutation relations (94) of the  $so(8)$  generators and the fermionic generators can be written in the  $7d$  form as

$$[V^{8I'}, O_{\alpha A'}] = -\frac{1}{2} \gamma^{I' A' B'} O_{\alpha B'}, \quad [V^{I' J'}, O_{\alpha A'}] = \frac{1}{2} \gamma^{I' J' A' B'} O_{\alpha B'} \quad (116)$$

and further transformed into the commutation relations of the superconformal generators (114) with the transformed  $so(8)$  generators (104)-(106), (108)

$$\begin{aligned} [T^a, Q_\mu^b] &= i\varepsilon^{abc} \bar{Q}_{\mu c}, \quad [T_a, \bar{Q}_{\mu b}] = -i\varepsilon_{abc} Q_\mu^c, \\ [T^a, S^{\mu b}] &= i\varepsilon^{abc} \bar{S}_c^\mu, \quad [T_a, \bar{S}_b^\mu] = -i\varepsilon_{abc} S^{\mu c}, \\ [\tilde{T}_a, Q_\mu^b] &= i\delta_a^b \bar{Q}_{\mu 4}, \quad [\tilde{T}^a, \bar{Q}_{\mu b}] = -i\delta_b^a Q_\mu^4, \\ [\tilde{T}_a, S^{\mu b}] &= i\delta_a^b \bar{S}_4^\mu, \quad [\tilde{T}^a, \bar{S}_b^\mu] = -i\delta_b^a S^{\mu 4}, \\ [\tilde{T}_a, Q_\mu^4] &= -i\bar{Q}_{\mu a}, \quad [\tilde{T}^a, \bar{Q}_{\mu 4}] = iQ_\mu^a, \\ [\tilde{T}_a, S^{\mu 4}] &= -i\bar{S}_a^\mu, \quad [\tilde{T}^a, \bar{S}_4^\mu] = iS^{\mu a}, \\ [H, Q_\mu^4] &= 2iQ_\mu^4, \quad [H, \bar{Q}_{\mu 4}] = -2i\bar{Q}_{\mu 4}, \quad [H, S^{\mu 4}] = 2iS^{\mu 4}, \quad [H, \bar{S}_4^\mu] = -2i\bar{S}_4^\mu, \\ [\tilde{V}_a^b, Q_\mu^c] &= \frac{i}{2} \delta_a^b Q_\mu^c - i\delta_a^c Q_\mu^b, \quad [\tilde{V}_a^b, \bar{Q}_{\mu c}] = -\frac{i}{2} \delta_a^b \bar{Q}_{\mu c} + i\delta_c^b \bar{Q}_{\mu a}, \\ [\tilde{V}_a^b, S^{\mu c}] &= \frac{i}{2} \delta_a^b S^{\mu c} - i\delta_a^c S^{\mu b}, \quad [\tilde{V}_a^b, \bar{S}_c^\mu] = -\frac{i}{2} \delta_a^b \bar{S}_c^\mu + i\delta_c^b \bar{S}_a^\mu, \\ [V_a^4, Q_\mu^b] &= -i\delta_a^b Q_\mu^4, \quad [V_4^a, \bar{Q}_{\mu b}] = i\delta_b^a \bar{Q}_{\mu 4}, \\ [V_a^4, S^{\mu b}] &= -i\delta_a^b S^{\mu 4}, \quad [V_4^a, \bar{S}_b^\mu] = i\delta_b^a \bar{S}_4^\mu, \\ [V_a^4, Q_\mu^4] &= -iQ_\mu^a, \quad [V_4^a, \bar{Q}_{\mu 4}] = i\bar{Q}_{\mu a}, \\ [V_a^4, S^{\mu 4}] &= -iS^{\mu a}, \quad [V_4^a, \bar{S}_4^\mu] = i\bar{S}_a^\mu. \end{aligned} \quad (117)$$

Note that the  $u(1)$  fiber generator  $H$  commutes with the unbroken supersymmetry generators. This can be considered as its defining property.

The commutation relations (95) of the  $sp(4)$  generators and the fermionic generators in the  $AdS_4$  basis acquire the form

$$[M_{0'm'}, O_{\alpha A'}] = -\Gamma_{m'\alpha}{}^\gamma \Gamma_\gamma^{5\beta} O_{\beta A'}, \quad [M_{m'n'}, O_{\alpha A'}] = -\frac{1}{2} \Gamma_{m'n'\alpha}{}^\beta O_{\beta A'}. \quad (118)$$

They can be cast into the commutation relations of the  $3d$  conformal generators (97) with the fermionic generators (114) and have the same form as for the  $osp(4|6)$  superalgebra modulo the incorporation of the broken supersymmetries generators

$$\begin{aligned} [D, Q_\mu^A] &= Q_\mu^A, & [D, \bar{Q}_{\mu A}] &= \bar{Q}_{\mu A}, \\ [M^{mn}, Q_\mu^A] &= \frac{1}{2} \sigma^{mn}{}_\mu{}^\nu Q_\nu^A, & [M^{mn}, \bar{Q}_{\mu A}] &= \frac{1}{2} \sigma^{mn}{}_\mu{}^\nu \bar{Q}_{\nu A}, \\ [K^m, Q_\mu^A] &= \sigma_{\mu\nu}^m S^{\nu A}, & [K^m, \bar{Q}_{\mu A}] &= \sigma_{\mu\nu}^m \bar{S}_A^\nu, \\ [D, S^{\mu A}] &= -S^{\mu A}, & [D, \bar{S}_A^\mu] &= -\bar{S}_A^\mu, \\ [M^{mn}, S^{\mu A}] &= -\frac{1}{2} S^{\nu A} \sigma^{mn}{}_\nu{}^\mu, & [M^{mn}, \bar{S}_A^\mu] &= -\frac{1}{2} \bar{S}_A^\nu \sigma^{mn}{}_\nu{}^\mu, \\ [P^m, S^{\mu A}] &= -\tilde{\sigma}^{m\mu\nu} Q_\nu^A, & [P^m, \bar{S}_A^\mu] &= -\tilde{\sigma}^{m\mu\nu} \bar{Q}_{\nu A}. \end{aligned} \quad (119)$$

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